Aberrations

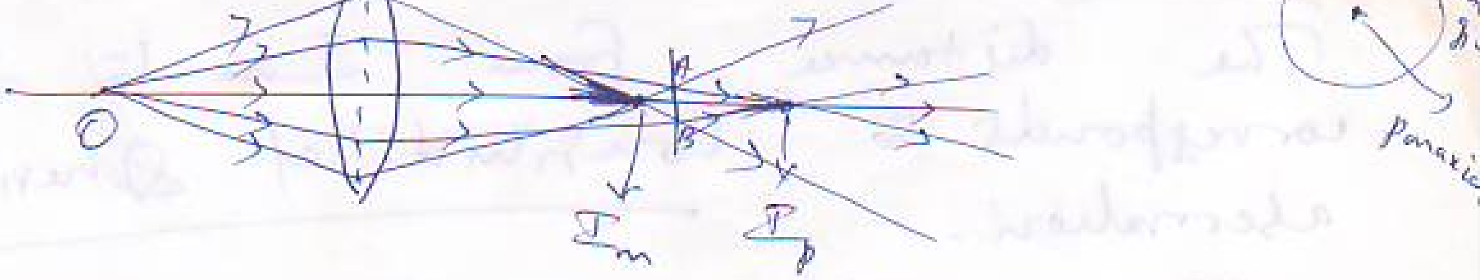
The refractive index & hence the focal length of a lens are different for different wavelengths.

The deviations of an image from the actual size, shape & position are called aberrations produced by a lens.

The aberrations produced by the variation of refractive index with wavelength of light are called Chromatic aberrations

~~Abber~~ Aberrations caused even if a monochromatic light is used are called monochromatic aberrations

Spherical Aberration in a lens.



In the figure O is the point object on the axis of the lens. I_p and I_m are the images formed by the paraxial and marginal rays respectively.

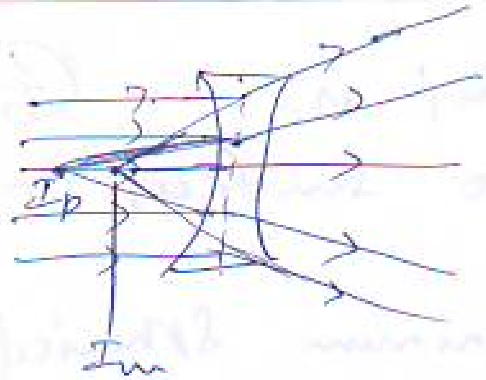
The paraxial rays form the image at a longer distance than the marginal rays. Hence the image is not very sharp because of two focussing / converging points.

But if a screen is placed at AB (where these two rays intersect) an image is formed as a circular patch (with diameter AB)

The circular image of diameter AB is called circle of least confusion. This corresponds to the position of best image.

The distance from I_m to I_p corresponds to longitudinal spherical aberration.

The radius of the circle AB gives the lateral spherical aberration.



The spherical aberration in a concave lens will be as shown in the figure.

The spherical aberration in a convex lens is positive.

The concave lens is negative.

Reducing Spherical aberration

1. Spherical aberration can be minimised by using stops. They reduce the effective lens aperture. But since the amount of light passing through the lens is reduced, the image will appear less bright.

2. The longitudinal spherical aberration (x) produced by a thin lens is given in terms of k where $k = R_1/R_2$

~~K~~ R_1 & R_2 → radii of curvature of the two surfaces of the lens.

The condition for minimum spherical aberration is $\frac{dx}{dK} = 0$

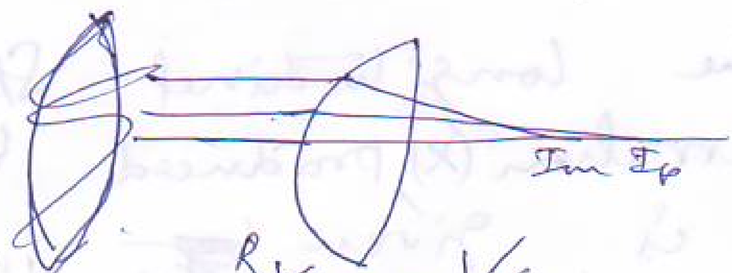
we can get, $K = \frac{R_1}{R_2} = \frac{\mu(2\mu - 1) - 4}{\mu(2\mu + 1)}$

If $\mu = 1.5$, $K = -\frac{1}{6}$

Thus for a biconcave lens, with ratio of radii of curvatures $\frac{1}{6}$ the spherical aberration will be minimum.

The radius of curvature of the surface facing the incident light is $\frac{1}{6}$ of the other surface.

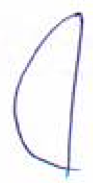
A lens for which $\frac{R_1}{R_2} = -\frac{1}{6}$ is called a crossed lens.



$\frac{R_1}{R_2} = -\frac{1}{6}$

The axial and marginal rays come to focus within a small distance.

3. Plano-convex lenses ~~e~~ are used to reduce spherical aberration



when the curve ~~face~~ surface faces the incident beam, the spherical aberration will be minimum.

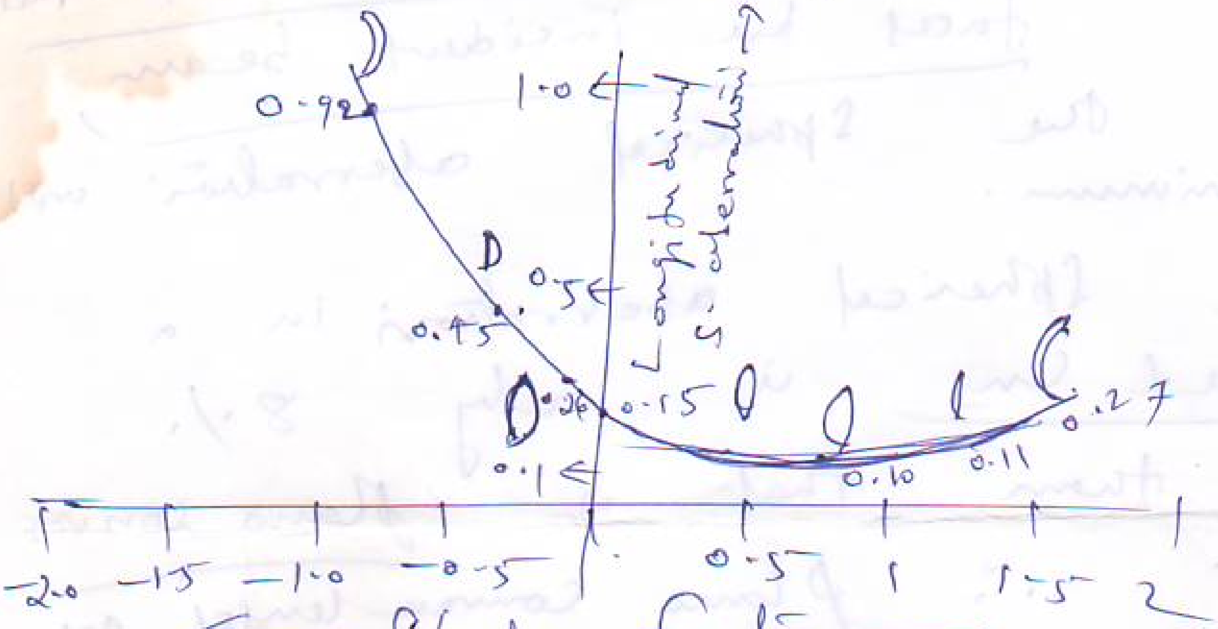
The spherical aberration in a crossed lens is only 8% less than that of plano convex lens. ∴ plano convex lenses are generally used to reduce spherical aberration.

Thus ~~..~~ ~~.~~ by choosing proper radii of curvature the S. aberration can be minimised. The shape factor of a lens is given by

$$q = \frac{R_2 + R_1}{R_2 - R_1}$$

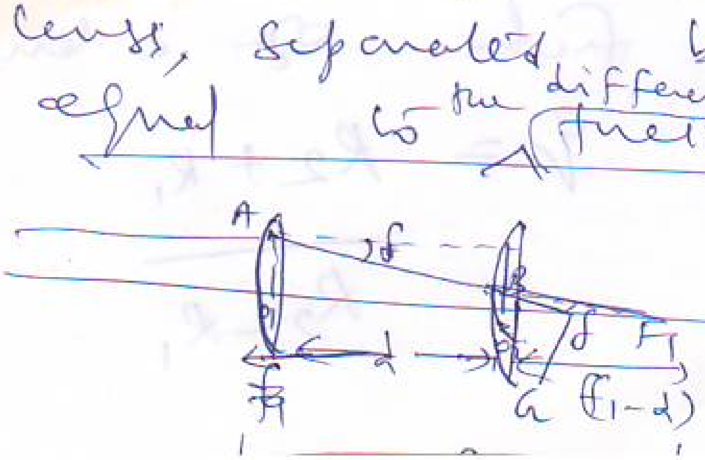
(6)

Figure shows lenses of same μ, f, & ρ (radius of lens) but different shape factors. The longitudinal S. aberration is plotted in the Y axis & shape factor in the x axis.



S. aberration for a double convex lens is minimum when the smaller radius surface faces the incident beam.

4. Spherical aberration can be minimised by using 2 plano convex lenses, separated by a distance equal to the difference in their focal lengths.



Let $\delta \rightarrow$ angle of deviation produced by each lens.

$$\angle BF_1G = \delta$$

$$\angle F_1BG = \delta$$

From ΔBGF_1 $\quad BG = GF_1$

~~BG =~~ $\quad O_2G = GF_1$

$$\therefore O_2G = \tfrac{1}{2}(O_2F_1) = \tfrac{1}{2}(f_1 - d)$$

For the II lens, F_1 is the virtual object and G is the real image

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_2}$$

$$\frac{1}{O_2G} - \frac{1}{O_2F_1} = \frac{1}{f_2}$$

$$\frac{2}{(f_1 - d)} - \frac{1}{(f_1 - d)} = \frac{1}{f_2}$$

$$\left(\frac{1}{f_1 - d}\right) = \frac{1}{f_2}$$

$$f_2 = (f_1 - d)$$

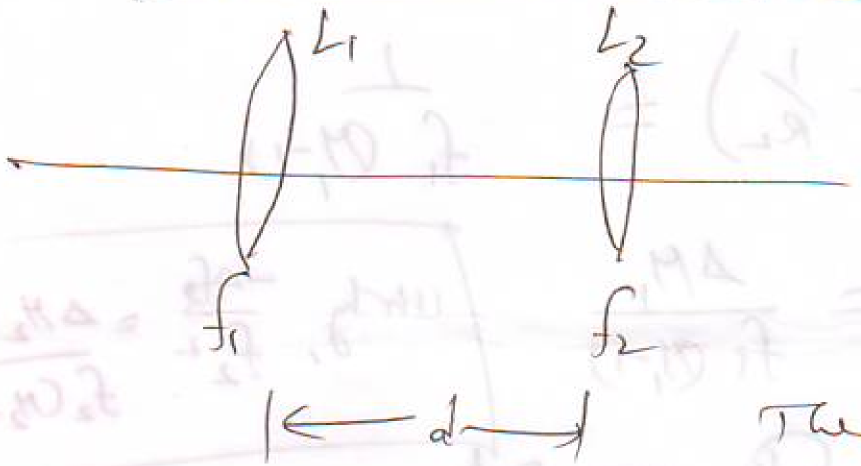
$$\boxed{d = f_1 - f_2}$$

Condition ii

The distance between the two lenses should be the difference in their [illegible]

Aberration (contd...)

in Two thin lenses separated by a distance.



L_1 and L_2 are two thin lenses separated by a distance d. The focal length of the combined lens is

$$F = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \rightarrow (1)$$

The focal length of first lens L_1 is

$$\frac{1}{f_1} = (\mu_1 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \rightarrow (2)$$

↳ refractive index.

The focal length of the second lens L_2 is,

$$\frac{1}{f_2} = (\mu_1 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \rightarrow (3)$$

Let $\Delta f_1 \rightarrow$ change in the focal length due to a change in

$\Delta \mu_1 \rightarrow$ " " " refractive index

" " " " " . . .

Differentiating (2) we get

$$-\frac{\Delta f_1}{f_1^2} = \Delta \mu_1\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

But $\dfrac{1}{f_1} = (\mu_1 - 1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$

$\therefore \left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right) = \dfrac{1}{f_1(\mu_1 - 1)}$

$\therefore \dfrac{-\Delta f_1}{f_1^2} = \dfrac{\Delta \mu_1}{f_1(\mu_1 - 1)}$ $\quad$ $\boxed{\text{IIIrly, } \dfrac{-\Delta f_2}{f_2^2} = \dfrac{\Delta \mu_2}{f_2(\mu_2 - 1)}}$

Differentiating (1), we get,

$$-\frac{\Delta F}{F^2} = -\frac{\Delta f_1}{f_1^2} - \frac{\Delta f_2}{f_2^2} + \frac{d}{f_2}\frac{\Delta f_1}{f_1^2} + \frac{d}{f_1}\frac{\Delta f_2}{f_2^2}$$

$$= \frac{\Delta \mu_1}{f_1(\mu_1 - 1)} + \frac{\Delta \mu_2}{f_2(\mu_2 - 1)} - \frac{d}{f_2}\frac{\Delta \mu_1}{f_1(\mu_1 - 1)} - \frac{d}{f_1}\frac{\Delta \mu_2}{f_2(\mu_2 - 1)}$$

$$= \frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} - \frac{d}{f_1 f_2}\left[\omega_1 + \omega_2\right]$$

$\omega_1, \omega_2 \rightarrow$ dispersive powers of the two lenses.

For the combined lens L_1 & L_2 to have the same focal length (minimum aberration) for blue & red colors $\dfrac{\Delta F}{F^2} = 0$, i.e., $\Delta F = 0$

$$\frac{d}{f_1 f_2}(\omega_1 + \omega_2) = \frac{\omega_1}{f_1} + \frac{\omega_2}{f_2}$$

③

$$d = \left(\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2}\right)\left(\frac{f_1 f_2}{(\omega_1+\omega_2)}\right)$$

$$\cancel{= \frac{\omega_1}{f_1} f_1 f_2}$$

$$\frac{d}{f_1 f_2} = \left(\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2}\right)\left(\frac{1}{\omega_1+\omega_2}\right)$$

$$d = \left(\frac{\omega_1}{\cancel{f_1}} \cancel{f_1} f_2 + \frac{\omega_2}{\cancel{f_2}} f_1 \cancel{f_2}\right)\left(\frac{1}{\omega_1+\omega_2}\right)$$

$$d = \left(\frac{\omega_1 f_2 + \omega_2 f_1}{\omega_1+\omega_2}\right)$$

If both lenses are made of the same material then $\omega_1 = \omega_2 = \omega$

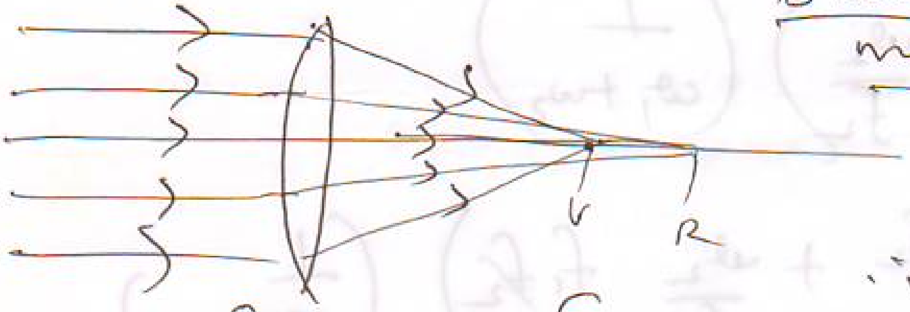
$$\therefore \quad d = \frac{\omega f_2 + \omega f_1}{\omega + \omega} = \frac{\cancel{\omega}(f_1+f_2)}{2\cancel{\omega}}$$

$$\boxed{d = \frac{f_1+f_2}{2}}$$

The chromatic aberration will be very small distance between the two thin lenses is the average focal length.

~~Chr~~ Chromatic aberration in a lens.

A parallel beam of white light is incident on a thin lens (Convex lens).



Blue light will be more refracted than Red light.

$\therefore$ Blue light will ~~focus~~ focus near the lens than the red light. Thus the image will appear colored.

The focal length of a thin lens is

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad (1)$$

$\Delta\mu \rightarrow$ change in μ due to change in wavelength (Color).

A change in μ will result in a change in the focal length Δf.

$$\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{f(\mu-1)}$$

Differentiating (1)

$$-\frac{\Delta f}{f^2} = \Delta\mu\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{\Delta\mu}{f(\mu-1)}$$

$$\therefore \Delta f = \frac{-\Delta\mu}{f(\mu-1)} \quad \text{— chromatic aberration}$$