Absent spectra with a diffraction Grating

The n^{th} order principal maximum of a diffraction grating is

$$(a+b)\sin\theta = n\lambda \quad (1)$$

Let the path difference be λ. Then the slit can be imagined to be made up of 2 halves. The path difference between any pair of corresponding points in the two halves is $\frac{\lambda}{2}$. This will result in zero intensity in that direction.

$\therefore$ The condition for a minimum for a single slit is

$$a\sin\theta = p\lambda \quad \text{where } p = 1, 2, 3 \ldots$$

already derived (2)

When condition (1) and (2) are simultaneously satisfied, the beams from all the slits reinforce each other and the resultant intensity will be zero

Hence the spectrum will be absent. (2)

Divide (1) by (2), we get

$$\frac{(a+b)\cancel{\sin\theta}}{a\cancel{\sin\theta}} = \frac{n\cancel{\lambda}}{p\cancel{\lambda}}$$

$$\boxed{\frac{(a+b)}{a} = \frac{n}{p}} \longrightarrow (3)$$

$\longrightarrow$ Conditions for n^{th} order spectrum to be absent.

(1) let $b = a$

From (3), $\frac{2\cancel{a}}{\cancel{a}} = n/p$, $\frac{n}{p} = 2$

$$\boxed{n = 2p}$$

$\therefore$ The 2^{nd}, 4^{th}, 6^{th} orders of spectrum will be missing corresponding to minima due to a single slit given by $p = 1, 2, 3$ ...etc

(2) when $b = 2a$,

$$\left(\frac{a+2a}{a}\right) = \frac{n}{p} = \frac{3\cancel{a}}{\cancel{a}} = \frac{n}{p}$$

The 3^{rd}, 6^{th}, 9^{th} order $\boxed{n = 3p}$

Dispersive power of a grating

The dispersive power of a grating is defined as the rate of change of the angle of diffraction with the wavelength of light.

If θ and $(\theta + d\theta)$ are the angles of diffraction corresponding to the wavelengths λ and $(\lambda + d\lambda)$ then, $\frac{d\theta}{d\lambda}$ is called the dispersive power of a grating

For a grating we have,

$$(a+b)\sin\theta = n\lambda \quad (1)$$

where $(a+b)$ is the grating element.

θ is the angle of diffraction for nth order spectrum.

Differentiate (1) w. r. to λ

$$(a+b)\cos\theta \frac{d\theta}{d\lambda} = n$$

$\frac{d\theta}{d\lambda}$ = Dispersive power.

$$\boxed{\frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta}}$$

(1) The dispersive power is directly $\propto$ to n, order of spectrum

(2) " " inversely proportional to the grating element $(a+b)$

$\because$ But $\frac{1}{a+b} = N$, no. of lines of the grating

$\therefore \frac{d\theta}{d\lambda} \propto N$

(3) The dispersive power is inversely $\propto$ to $\cos\theta$.

Overlapping of Spectral lines.

If the light incident on a grating contains a large no. of wavelengths, then the spectral lines of shorter wavelengths and higher order can overlap with the spectral lines of longer wavelength and lower order.

(eg) The ~~thired~~ third order $(n=3)$ red line $(\lambda = 7000\,Å)$ fourth order $(n=4)$ green line $(\lambda = 5250\,Å)$ and fifth order $(n=5)$ violet line $(\lambda = 4200\,Å)$ will all occur in the same direction θ.

Because, $(a+b)\sin\theta = 3 \times 7000 \times 10^{-10} = 4 \times 5250 \times 10^{-10} = 5 \times 4200 \times 10^{-10}$