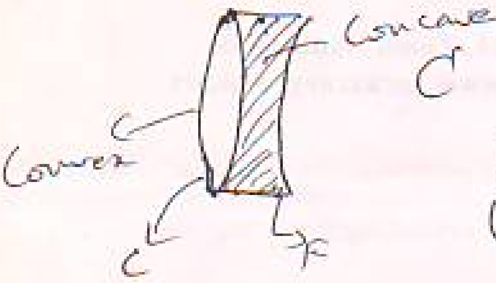
Achromatism of two thin lenses placed in contact

Consider two lenses placed in contact.



C is a convex lens of Crown glass and F is a concave lens of ~~[illegible]~~ flint glass.

Let μ_b, μ, μ_r are the refractive indices for blue, yellow and red rays of light for the crown glass lens

Let μ'_b, μ', μ'_r are refractive indices for blue, yellow and red rays of light for the flint glass.

f_b, f, f_r and f'_b, f', f'_r are the corresponding focal lengths for blue, yellow and red rays of light for these two lenses.

Let ω and ω' are the dispersive powers of the two lenses.

(2)

We have, $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ (1)

$$\frac{1}{f_b} = (\mu_b - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad (2)$$

$$\frac{1}{f_r} = (\mu_r - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad (3)$$

$$\frac{1}{f'} = (\mu' - 1)\left(\frac{1}{R_1'} - \frac{1}{R_2'}\right) \quad (4)$$

$$\frac{1}{f_b'} = (\mu_b' - 1)\left(\frac{1}{R_1'} - \frac{1}{R_2'}\right) \quad (5)$$

$$\frac{1}{f_r'} = (\mu_r' - 1)\left(\frac{1}{R_1'} - \frac{1}{R_2'}\right) \quad (6)$$

From (1), $\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{(\mu - 1) f}$ (7)

From (4), $\left(\frac{1}{R_1'} - \frac{1}{R_2'}\right) = \frac{1}{(\mu' - 1) f'}$ (8)

Substitute (7) in (2) & (3)

$$\frac{1}{f_b} = \frac{(\mu_b - 1)}{(\mu - 1) f}, \qquad \frac{1}{f_r} = \frac{(\mu_r - 1)}{(\mu - 1) f}$$

Substitute (8) in (5) & (6)

$$\frac{1}{f_b'} = \frac{(\mu_b' - 1)}{(\mu' - 1) f'}, \qquad \frac{1}{f_r'} = \frac{(\mu_r' - 1)}{(\mu' - 1) f'}$$

~~Let F_b and F_r are the focal lengths of the~~

Let F_b is the combined focal length for blue rays of the two lenses

Let F_r - " - " - red rays " - " -

$$\frac{1}{F_b} = \frac{1}{f_b} + \frac{1}{f_b'} = \frac{(\mu_b - 1)}{(\mu - 1)f} + \frac{(\mu_b' - 1)}{(\mu' - 1)f'}$$

$$\frac{1}{F_r} = \frac{1}{f_r} + \frac{1}{f_r'} = \frac{(\mu_r - 1)}{(\mu - 1)f} + \frac{(\mu_r' - 1)}{(\mu' - 1)f'}$$

For the combined lens to be achromatic, the focal lengths F_b and F_r must be equal.

$F_b = F_r$, or $\frac{1}{F_b} = \frac{1}{F_r}$

$$\therefore \frac{(\mu_b - 1)}{(\mu - 1)f} + \frac{(\mu_b' - 1)}{(\mu' - 1)f} = \frac{(\mu_r - 1)}{(\mu - 1)f} + \frac{(\mu_r' - 1)}{(\mu' - 1)f'}$$

$$\frac{(\mu_b - 1)}{(\mu - 1)f} + \frac{(\mu_b' - 1)}{(\mu' - 1)f} - \frac{(\mu_r - 1)}{(\mu - 1)f} - \frac{(\mu_r' - 1)}{(\mu' - 1)f'} = 0$$

$$\frac{(\mu_s - 1)}{(\mu - 1) f} - \frac{(\mu_r - 1)}{(\mu - 1) f} + \frac{(\mu_s' - 1)}{(\mu' - 1) f'} - \frac{(\mu_r' - 1)}{(\mu' - 1) f'} = 0$$

$$\frac{(\mu_s - \mu_r)}{(\mu - 1) f} + \frac{(\mu_s' - \mu_r')}{(\mu' - 1) f'} = 0$$

$$\frac{\omega}{f} + \frac{\omega'}{f'} = 0$$

$$\therefore \frac{\omega}{f} = -\frac{\omega'}{f'}$$

$$f' = -\frac{\omega'}{\omega} f$$

ω' & ω → positive.

Hence, f' is +ve if f is negative

f' is −ve if f is positive.