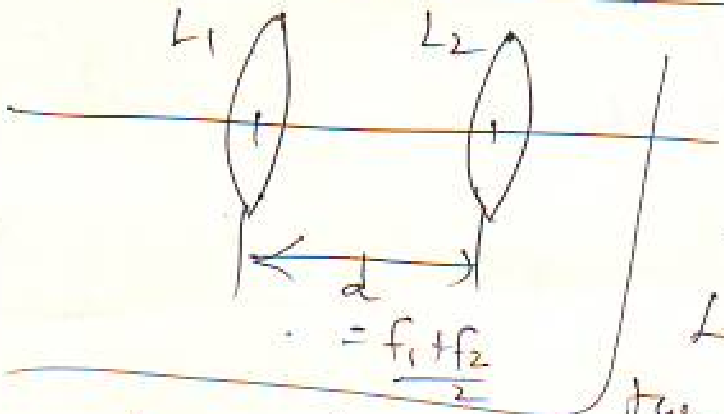
Achromatism of two thin lenses separated by a finite distance

(1)



Let L_1 and L_2 are two thin lenses separated by a distance d. Let f_1 and f_2 be the ~~of~~ focal lengths of the two lenses.

Let the two lenses are made of the same material.

If μ_b and μ_r are the refractive indices for the blue and red rays of light. Let μ be the mean refractive index.

Let f_b and f_r are the focal lengths for the blue and red rays of light for the first lens.

Let f_b' and f_r' — — — —

— — — second lens.

now the focal length of the combination is (for mean rays)

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$$\frac{1}{F_r} = \frac{1}{f_r} + \frac{1}{f_r'} - \frac{d}{f_r f_r'} \quad \text{(for red rays)}$$

$$\frac{1}{F_b} = \frac{1}{f_b} + \frac{1}{f_b'} - \frac{d}{f_b f_b'} \quad \text{(for blue rays)}$$

we have, $\frac{1}{f_r} = \frac{(\mu_r - 1)}{(\mu - 1) f_1}$, $\frac{1}{f_r'} = \frac{(\mu_r - 1)}{(\mu - 1) f_2}$

$\frac{1}{f_b} = \frac{(\mu_b - 1)}{(\mu - 1) f_1}$, $\frac{1}{f_b'} = \frac{(\mu_b - 1)}{(\mu - 1) f_2}$

$$\frac{1}{F_r} = \frac{(\mu_r - 1)}{(\mu - 1) f_1} + \frac{(\mu_r - 1)}{(\mu - 1) f_2} - \frac{(\mu_r - 1)^2}{(\mu - 1)^2} \frac{d}{f_1 f_2}$$

$$\frac{1}{F_b} = \frac{(\mu_b - 1)}{(\mu - 1) f_1} + \frac{(\mu_b - 1)}{(\mu - 1) f_2} - \frac{(\mu_b - 1)^2}{(\mu - 1)^2} \frac{d}{f_1 f_2}$$

For the combined lens to be achromatic

$$F_r = F_b$$

$$\therefore \frac{(\mu_r - 1)}{(\mu - 1) f_1} + \frac{(\mu_r - 1)}{(\mu - 1) f_2} - \frac{(\mu_r - 1)^2}{(\mu - 1)^2} \frac{d}{f_1 f_2}$$

$$= \frac{(\mu_b - 1)}{(\mu - 1) f_1} - \frac{(\mu_b - 1)}{(\mu - 1) f_2} - \frac{(\mu_b - 1)^2}{(\mu - 1)^2} \frac{d}{f_1 f_2}$$

$$\frac{(\mu_r-1)}{(\mu-1)}\left[\frac{1}{f_1}+\frac{1}{f_2}\right]-\frac{(\mu_r-1)^2}{(\mu-1)^2}\frac{d}{f_1f_2}$$

$$=\frac{(\mu_b-1)}{(\mu-1)}\left[\frac{1}{f_1}+\frac{1}{f_2}\right]-\frac{(\mu_b-1)^2}{(\mu-1)^2}\frac{d}{f_1f_2}$$

$$\left[\left(\frac{\mu_r-1}{\mu-1}\right)-\frac{(\mu_b-1)}{(\mu-1)}\right]\left(\frac{1}{f_1}+\frac{1}{f_2}\right)$$

$$=\frac{(\mu_r-1)^2}{(\mu-1)^2}\frac{d}{f_1f_2}-\frac{(\mu_b-1)^2}{(\mu-1)^2}\frac{d}{f_1f_2}$$

$$\frac{(\mu_r-\mu_b)}{(\mu-1)}\left(\frac{1}{f_1}+\frac{1}{f_2}\right)=\frac{\left((\mu_r-1)^2-(\mu_b-1)^2\right)}{(\mu-1)^2}\frac{d}{f_1f_2}$$

$$=\frac{d}{(\mu-1)^2f_1f_2}\left[\mu_r^2\cancel{+1}-2\mu_r-\mu_b^2\cancel{-1}+2\mu_b\right]$$

$$=\frac{d}{(\mu-1)^2f_1f_2}\left[(\mu_r^2-\mu_b^2)-2(\mu_r-\mu_b)\right]$$

$$=\frac{d}{(\mu-1)^2f_1f_2}\left[(\mu_r+\mu_b)(\mu_r-\mu_b)-2(\mu_r-\mu_b)\right]$$

$$=\frac{d}{(\mu-1)^2f_1f_2}\left[(\mu_r-\mu_b)\left((\mu_r+\mu_b)-2\right)\right]$$

$\mu_b + \mu_r = 2\mu$ $\quad \mu_r^2 \cancel{(f_1)} - 2\mu_r$

$\mu_b - \mu_r$ $\quad \cancel{2f_1}\,\mu_b^2\,\cancel{(f_2)} + 2\mu_b)$

$= 2\mu - \mu_r - \mu_r$

$= 2\mu - 2\mu_r\,(\mu_r^2 - \mu_b^2) - 2(\mu_r - \mu_b)$

$$= \frac{d}{(\mu-1)^2 f_1 f_2}\left[\mu_r - \mu_b\right]\left[2\mu - 2\right]$$

(let $\mu_r + \mu_b = 2\mu$)

$$= \frac{(\mu_r - \mu_b)\,d}{(\mu-1)^2 f_1 f_2}\left[2\,\cancel{(\mu-1)}\right] = \frac{2(\mu_r - \mu_b)\,d}{(\mu-1)\,f_1 f_2}$$

$$\therefore \left(\frac{\mu_r - \mu_b}{\mu - 1}\right)\left(\frac{1}{f_1} + \frac{1}{f_2}\right) = \frac{2(\mu_r - \mu_b)\,d}{(\mu-1)\,f_1 f_2}$$

$$\therefore \frac{1}{f_1} + \frac{1}{f_2} = \frac{2d}{f_1 f_2}$$

$$\frac{f_1 + f_2}{\cancel{f_1 f_2}} = \frac{2d}{\cancel{f_1 f_2}}$$

$$f_1 + f_2 = 2d$$

$$\boxed{d = \frac{f_1 + f_2}{2}}$$