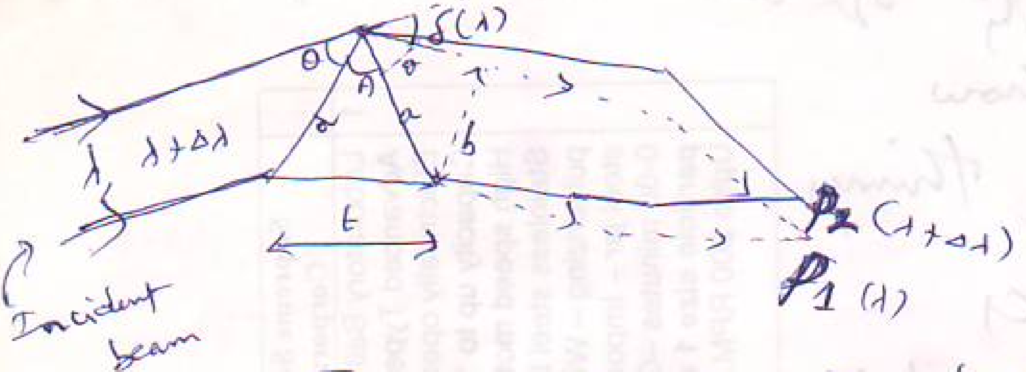
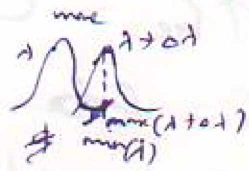
Resolving power of a prism

(See next page first) ①





The angle of the prism is A.

δ is the angle of minimum deviation.

The refractive index of the prism is

$$\mu = \frac{\sin\left(\frac{A+\delta(\lambda)}{2}\right)}{\sin\left(\frac{A}{2}\right)} \quad (1)$$

The images corresponding to λ and $\lambda+\Delta\lambda$ are at P_1 and P_2.

$\Delta\lambda \to$ too small and the minimum deviation positions are same for both wavelengths λ and $\lambda+\Delta\lambda$.

($b\sin\alpha = m\lambda$ single slit diffraction)

The diffraction lines from λ & $\lambda+\Delta\lambda$ are just resolved when the first diffraction ~~maximum~~ minimum of λ fall at the central maximum of $\lambda+\Delta\lambda$.

$$\therefore \Delta\lambda \simeq \frac{\lambda}{b} \quad (2) \quad \left(\frac{\lambda}{\Delta\lambda} = \text{resolving power} = R = b\right)$$

Differentiate ①

$$\frac{d\mu}{d\lambda} = \frac{1}{\sin(A/2)} \cos\left[\frac{A+\delta(\lambda)}{2}\right] \frac{1}{2} \frac{d\delta}{d\lambda}$$

Resolving power (2)

The capacity of an instrument to ~~cto~~ show two closely things separately is called resolution.

The ability of an optical instrument to resolve the images of two nearby points is called its resolving power.

$\therefore$ $\cancel{\Delta\lambda}$

$$\Delta\delta = \frac{2\sin\left(\frac{A}{2}\right)}{\cos\left(\frac{A+\delta(\lambda)}{2}\right)}\frac{dn}{d\lambda}\Delta\lambda$$

From figure,

$$\theta = \frac{1}{2}\left[\pi - (A+\delta)\right] \rightarrow \theta = 90 - \left(\frac{A+\delta}{2}\right)$$

$$\sin\theta = \frac{b}{a} = \cos\left(\frac{A+\delta(\lambda)}{2}\right)$$

From figure

Again $\sin\left(\frac{A}{2}\right) = \frac{t/2}{a}$

Thus $\Delta\delta = 2\,\frac{t/2}{a\,(b/a)}\,\frac{dn}{d\lambda}\Delta\lambda$

$$\Delta\delta \cong \frac{t}{b}\frac{dn}{d\lambda}\Delta\lambda$$

($\Delta\lambda = \frac{\lambda}{b}$ eqn(2)
$b\Delta\lambda \cong \lambda$
$b\Delta\delta = \lambda$)

$\cancel{\Delta\delta = \frac{t}{b}\left(\frac{dn}{d\lambda}\right)\left(\frac{\lambda}{b}\right)}$

~~The resolving power~~ $\cancel{R = \frac{\lambda}{\Delta\lambda}}$

$$\cancel{\therefore \frac{\lambda}{\Delta\lambda} = b = \frac{t}{\Delta\delta}\left(\frac{dn}{d\lambda}\right)\Delta\lambda = \frac{t}{\Delta\delta}\left(\frac{dn}{d\lambda}\right)\frac{\lambda}{b}}$$

$$\Delta\lambda = \frac{\Delta\delta}{\frac{t}{b}\left(\frac{dn}{d\lambda}\right)} \; ; \quad \frac{1}{\Delta\lambda} = \left(\frac{t}{b}\right)\left(\frac{dn}{d\lambda}\right)\left(\frac{1}{\Delta\delta}\right)$$

$$\boxed{R = \frac{\lambda}{\Delta\lambda} = t\left(\frac{dn}{d\lambda}\right)} \quad \because (b\Delta\delta = \lambda)$$

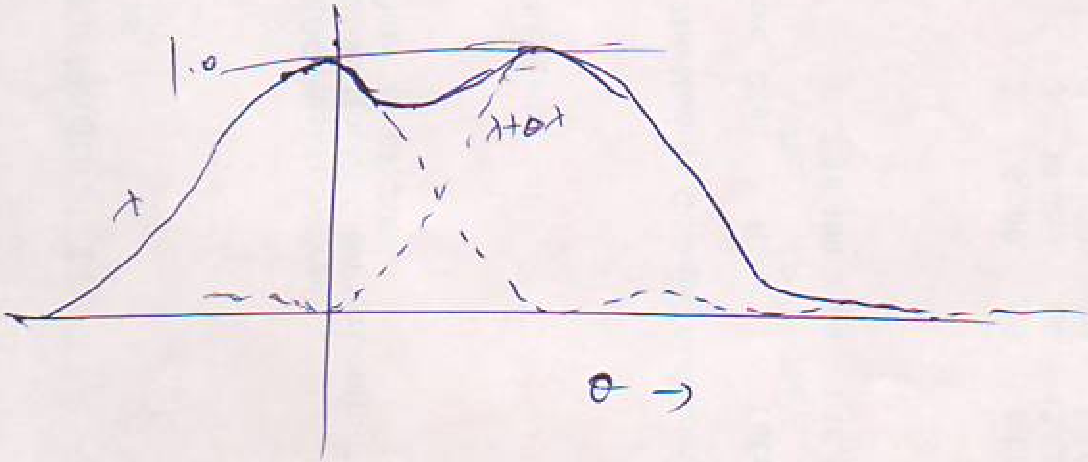
Resolving power of a plane diffraction grating

The resolving power is $R = \frac{\lambda}{\Delta\lambda}$

$\Delta\lambda \rightarrow$ Separation of two wavelengths which the grating can resolve.

When $\Delta\lambda$ is small, R will be larger.

If the principal maximum corresponding to the wavelength $\lambda + \Delta\lambda$ falls on the first minimum of the wavelength λ then the two wavelengths λ and $\lambda + \Delta\lambda$ are said to be just resolved.



$$\cancel{\therefore \; d \sin\theta = m(\lambda + \Delta\lambda)}$$

$$\cancel{d \sin\theta = m\lambda + m\Delta\lambda}$$

~~Where N is the total no. of lines in the grating.~~

$$R = \cancel{\frac{\lambda}{\Delta\lambda}} =$$

If the common diffraction angle is θ, then for mth order spectrum, the two wavelengths will be just resolved if the following 2 equations are simultaneously satisfied.

$$d \sin\theta = m(\lambda + \Delta\lambda) \quad (1)$$

$$d \sin\theta = m\lambda + \frac{\lambda}{N}$$

$N \rightarrow$ no. of lines in the grating

$$\therefore \quad m(\lambda + \Delta\lambda) = m\lambda + \frac{\lambda}{N}$$

$$\cancel{m\lambda} + m\Delta\lambda = \cancel{m\lambda} + \frac{\lambda}{N}$$

$$m\Delta\lambda = \frac{\lambda}{N}$$

$$\boxed{R = \frac{\lambda}{\Delta\lambda} = mN}$$

R depends on the number of lines in the grating.